Random Dynamics and Memory: Structure within Chaos?

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Abstract:

We give an overview of the dynamics and long-term evolution of probabilistic models with finite memory. Such models are widely used to analyze dynamical systems whose evolution is influenced by random fluctuations and past history. These models are important in diverse areas such as signal processing, option-pricing, economic and labor models, aircraft dynamics, materials science and population dynamics.

The dynamics of random systems with memory is treated at two different levels: distributional and pathwise. On the distributional level, the dynamics is described in terms of semigroup theory; on the pathwise level, the ergodic theory of stochastic flows is used to characterize the long-term behavior of the random evolution in the models near their statistical equilibria. This characterization is expressed via the existence of random flow-invariant stable and unstable manifolds near statistical equilibria. Thus, in spite of its inherent randomness, the system dynamics generically exhibits a definite structure near its statistical equilibria!

A key idea behind the analysis is to encode the system memory by "slicing" each random evolution path at any time. Each slice is viewed as repersenting an infinite-dimensional "state" of the random dynamical system at a particular moment. The existence of stable and unstable manifolds is established using a combination of diverse techniques from probability, stochastic calculus, stochastic differential equations, functional analysis, ergodic theory and dynamical systems.

The figure below shows the pathwise stability structure of the random dynamics near a statistical equilibrium.