Finite and Infinite-Dimensional Stochastic Dynamical Systems
Highlights

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Random Dynamics:

The project focused on qualitative and long-term behavior of a large class of probabilistic models known as stochastic differential equations. These equations are widely used by scientists and engineers. Of special interest is a class of models that are used in physics, engineering and biology in order to analyze dynamical systems whose evolution is influenced by random fluctuations and past history. Such models are very important in a variety of diverse areas such as signal processing, stock market fluctuations, economic and labor models, aircraft dynamics, materials with memory, population dynamics and fluid flow. Salah Mohammed, the principal investigator, used the most current probabilistic techniques in order to develop a deeper understanding of these models. The outcome of the research in this project characterized the long-term behavior of the random evolution in the models near their statistical equilibria. More specifically, the principal investigator, developed the following strategy in order to study the evolution of random dynamical systems with memory:

- Encode the system memory by "slicing" each random evolution path. Each slice is viewed as reperesenting the "state" of the random dynamical system at any particular time. The collection of all possible slices is referred to as the *state space*.

- The state space is furnished with algebraic operations (*addition, subtraction, scaling*) and a measure of *distance* and *angle* between states. The state space is *large*, with *infinitely many dimensions*.

- View the time-evolution of each slice within the state space.

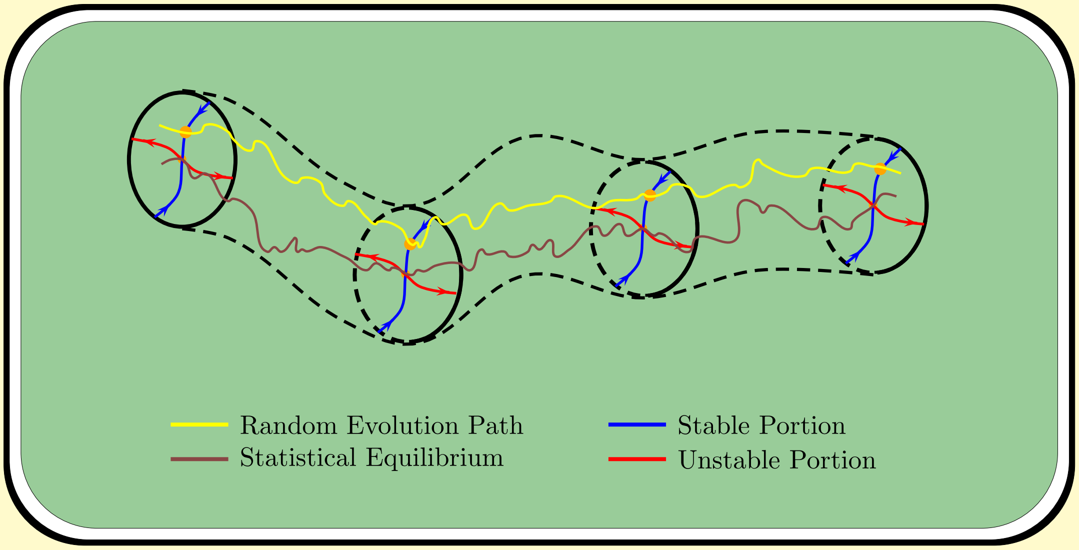
- Characterize the stability of statistical *equilibrium states*.

- Existence of stable/unstable "smooth portions" of the state space near equilibria. Thus, in spite of its apparent chaotic setting, the dynamics generically exhibits a definite structure near its statistical equilibria.

A similar stability structure near equilibria is established for a large class of models called stochastic partial differential equations. Such models are ubiquitous in the study of heat flow, fluid dynamics and modelling climate change.

The figure below shows the stability structure of the random dynamics near a statistical equilibrium. Further details are provided in Mohammed's web-site:

http://sfde.math.siu.edu/recentpub.html



Option-Pricing Models in Finance:

Numerical algorithms were developed whereby market data were used to test option-pricing models in which the stock price is governed by its past history. An explicit delayed Black-Scholes formula for pricing European options when the underlying stock price follows a non-linear random dynamical system with memory. The proposed delayed Black-Scholes model is sufficiently flexible to fit real market data, and is yet simple enough to allow for a closed-form representation of the option price as well as an explicit formula for the investment strategy.

Impact on Other Disciplines:

The research solidifies important connections with other areas of modern mathematical research, in particular the theory of dynamical systems, partial differential equations, numerical analysis and differential geometry. The results of the project are being compiled in a research monograph targeting graduate students majoring in mathematics, engineering, and finance.